Orbital Motion, Harvard Physics Circle

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Review

Kepler's laws are the following:

(I) The trajectories of objects under the influence of the Sun's gravity are conic sections.

(II) The trajectories sweep out area at a constant rate with the origin at the Sun.

(III) If the trajectory is closed (elliptical), then the period T is $\sqrt{\frac{4\pi^2}{GM}a^3}$, where a is the semimajor axis of the ellipse.

Another crucially important fact is that the energy of an orbit with semimajor axis a is

$$E = -\frac{GMm}{2a}.$$

Kepler (II) follows from angular momentum conservation about the Sun, so it holds for any central force.

Problems

Problem 1. [Adapted from F=ma 2013/13] There is a ring outside of Saturn. In order to distinguish if the ring is actually a part of Saturn or is instead part of the satellites of Saturn, we need to know the relation between the velocity v of each layer in the ring and the distance R of the layer to the center of Saturn. What is the relation if the ring is a part of Saturn, and what is the relation if the ring is part of the satellites of Saturn?

Problem 2. [F=ma 2013/18] See the end of this document for problem statement.

Problem 3. [F=ma 2014/22] A body of mass M and a body of mass $m \ll M$ are in circular orbits about their center of mass under the influence of their mutual gravitational attraction

to each other. The distance between the bodies is R, which is much larger than the size of either body.

A small amount of matter $\delta m \ll m$ is removed from the body of mass m and transferred to the body of mass M. The transfer is done in such a way so that the orbits of the two bodies remain circular, and remain separated by a distance R. Which of the following statements is correct?

(A) The gravitational force between the two bodies increases.

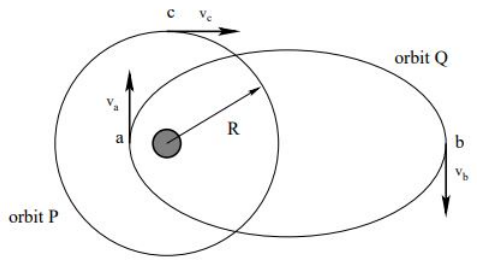
(B) The gravitational force between the two bodies remains constant.

(C) The total angular momentum of the system increases.

(D) The total angular momentum of the system remains constant.

(E) The period of the orbit of two bodies remains constant.

Problem 4. [F=ma 2012/25]



Consider the two orbits around the sun shown below. Orbit P is circular with radius R, orbit Q is elliptical such that the farthest point b is between $2R$ and $3R$, and the nearest point a is between $R/3$ and $R/2$. Consider the magnitudes of the velocity of the circular orbit v_c, the velocity of the comet in the elliptical orbit at the farthest point v_b, and the velocity of the comet in the elliptical orbit at the nearest point v_a. Which of the following rankings is correct?

(A) $v_b > v_c > 2v_a$

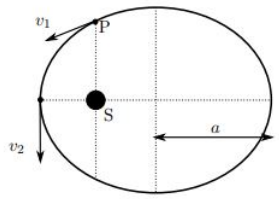
(B) $2v_c > v_b > v_a$

(C) $10v_b > v_a > v_c$

(D) $v_c > v_a > 4v_b$

(E) $2v_a > \sqrt{2}v_b > v_c$

Problem 5. [Adapted from F=ma 2017/25]



A planet orbits around a star S, as shown in the figure. The semi-major axis of the orbit is a. The perigee, namely the shortest distance between the planet and the star is $0.5a$. When the planet passes point P (on the line through the star and perpendicular to the major axis), its speed is v_1. What is its speed v_2 when it passes the perigee?

Problem 6. [[PPP] 88] A rocket is launched from and returns to a spherical planet of radius R so that its velocity vector on return is parallel to its velocity vector at launch. The angular separation at the center of the planet between the launch and arrival points is θ. How long does the flight take, if the period of a satellite flying around the planet just above its surface is T_0?

Problem 7. [Classical] Let R be the radius of the Earth. An object is dropped from a height R above the Earth's surface. How long does it take to reach the surface?

More Practice

- Do USAPhO 2012 A4: https://www.aapt.org/physicsteam/2013/upload/E3-2-3.pdf
- Do USAPhO 2015 B1: https://www.aapt.org/physicsteam/2015/upload/E3-2-5.pdf

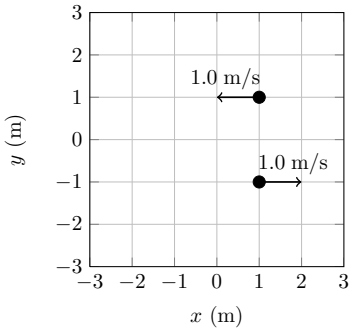
References

[M1] Morin, David J. *Problems and Solutions in Introductory Mechanics.*

[M2] Morin, David J. *Introduction to Classical Mechanics: With Problems and Solutions.*

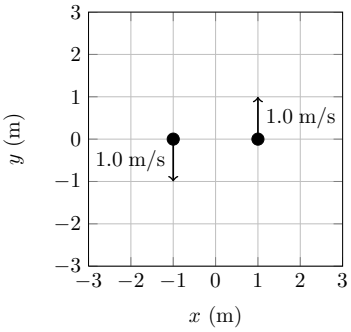
[PPP] Gnädig, Peter and Honyek Gyula and Riley, Ken. *200 Puzzling Physics Problems With Hints and Solutions.*

18. Two point particles, each of mass 1 kg, begin in the state shown below.

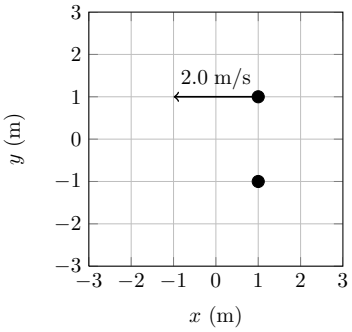


The system evolves through internal forces only. Which of the following could be the state after some time has passed?

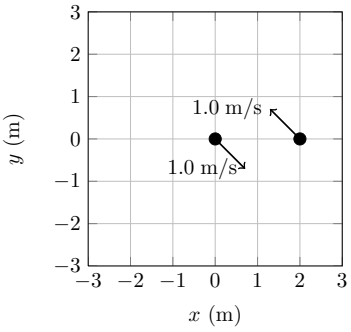
(A)



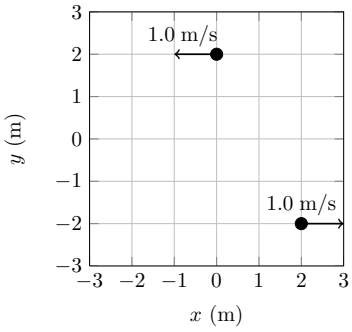
(B)



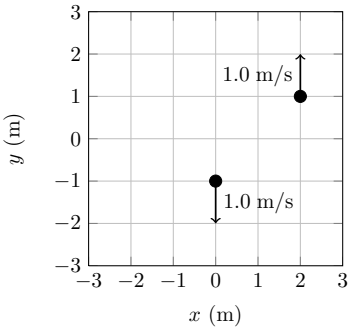
(C)



(D)



(E)



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