Statics, Harvard Physics Circle

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Review

Consider an object with forces $\mathbf{F}_1, \ldots, \mathbf{F}_N$ at locations $\mathbf{r}_1, \ldots, \mathbf{r}_N$. If this object is completely static, then

$$\sum_{i=1}^{N} \mathbf{F}_i = 0 \tag{1}$$

and

$$\sum_{i=1}^{N} \mathbf{r}_i \times \mathbf{F}_i = 0. \tag{2}$$

The quantity in (2) is known as the *torque*.

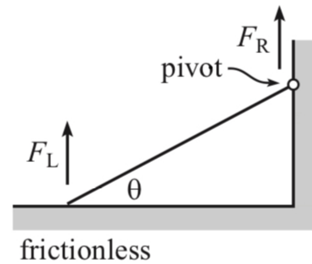
Problem 1. An equation with physical content should not be dependent on our arbitrary choice of origin. Equation (2) appears to violate this rule. Explain what is going on.

Remark 1. Equations (1) and (2) don't just apply to single objects, but can also be applied to any static system as a whole (be careful to note that only *external* forces count now). Keep this in mind when solving problems. Picking the right system can often be crucial for making problems simpler, or sometimes even tractable!

Remark 2. A corollary of Problem 1 is that it suffices to check (2) for only one choice of origin. Similar to picking the right system, picking the right system can often be crucial! Usually you want as many forces to cancel out either due to $\mathbf{r} = 0$ or $\mathbf{r} \parallel \mathbf{F}$. See Problem 8 for an extreme example.

Problems

Problem 2. [[M1] Multiple Choice 9.2]



One end of a stick with mass m and length l is pivoted on a wall, and the other end rests on a frictionless floor, as shown in the diagram. Let F_L and F_R be the vertical forces on the left and right ends of the stick, respectively. Then

(A) $F_L > F_R$

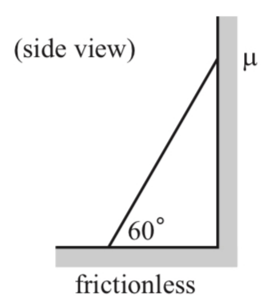
(B) $F_L < F_R$

(C) $F_L = F_R = mg/2$

(D) $F_L = F_R = mg$

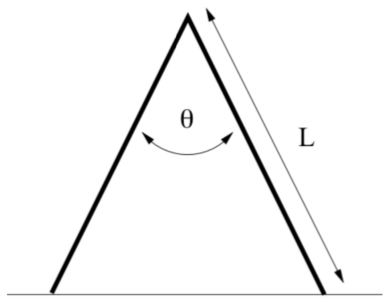
(E) $F_L = F_R = mg\cos\theta$

Problem 3. [[M1] Multiple Choice 9.3]



A ladder leans against a wall at a 60° angle, as shown in the diagram. The floor is frictionless, but there is friction with the wall. Assume that the coefficient of friction is large (say, $\mu = 10$). Is it possible for this setup to be static?

Problem 4. [F=ma 2013/4]



The sign shown below consists of two uniform legs attached by a frictionless hinge. The coefficient of friction between the ground and the legs is μ. Which of the following gives the maximum value of θ such that the sign will not collapse?

(A) $\sin\theta = 2\mu$

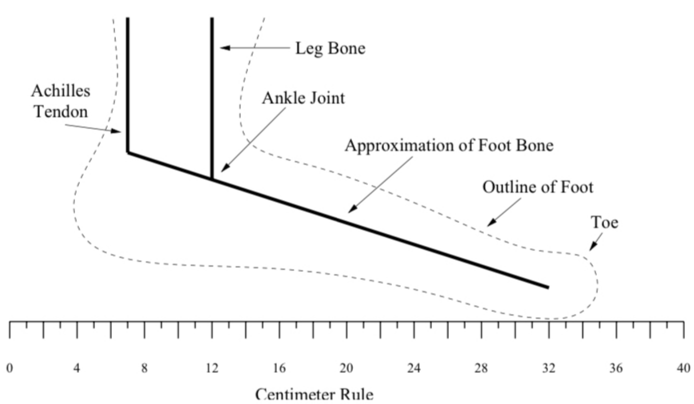
(B) $\sin\theta/2 = \mu/2$

(C) $\tan\theta/2 = \mu$

(D) $\tan\theta = 2\mu$

(E) $\tan\theta/2 = 2\mu$

Problem 5. [F=ma 2013/22]



A simplified model on the foot is shown. When a student of mass $m = 60$ kg stands on a single toe, the tension T in the Achilles Tendon is closest to

(A) $T = 600$ N

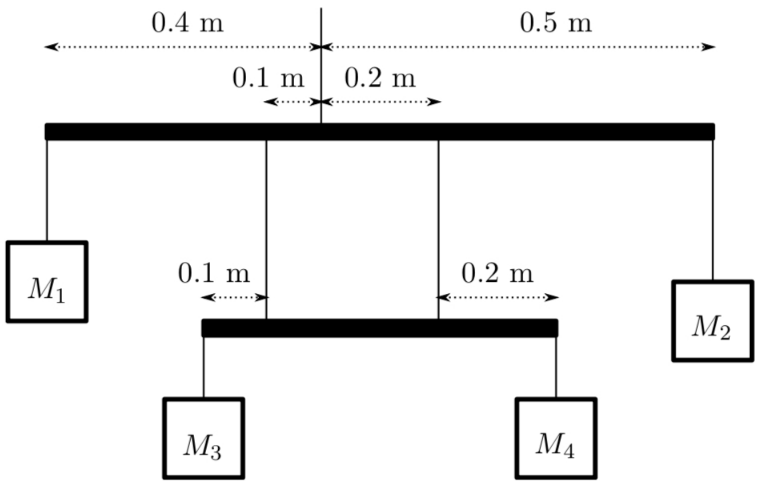
(B) $T = 1200$ N

(C) $T = 1800$ N

(D) $T = 2400$ N

(E) $T = 3000$ N

Problem 6. [F=ma 2017/6]



In the mobile below, the two cross beams and the seven supporting strings are massless. The hanging objects are $M_1 = 400$ g, $M_2 = 200$ g,and $M_4 = 500$ g. What is the value of M_3 for the system to be in static equilibrium?

(A) 300 g

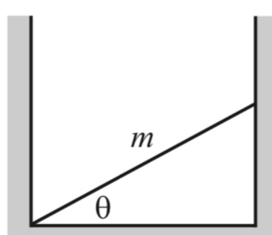
(B) 400 g

(C) 500 g

(D) 600 g

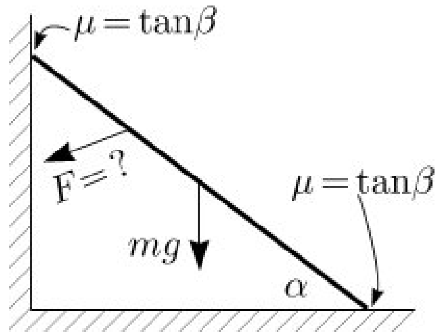
(E) 700 g

Problem 7. [[M1] Problem 9.3]



A uniform stick with mass m is placed in a frictionless well. The angle the stick makes with the horizontal is θ, as shown in the diagram. What are the forces the well exerts on the stick at its two ends?

Problem 8. [Physics Cup 2012/10] A thin rod of mass m is placed into a corner formed by a vertical wall and a horizontal floor so that the rod forms an angle α with the floor and is perpendicular to the line where the floor and the wall meet. The static coefficient of friction of the rod against the wall and against the floor is $\mu = \tan\beta$, which is not large enough for keeping the current position as long as the only forces applied to the rod are the normal and and friction forces applied to its endpoints, and the gravity force mg applied to its centre. What is the minimal additional force F needed for maintaining the current position of the rod (assuming that its direction and application point are optimal)? Express your answer in terms of m, g, α, and β.



More Practice

- Do all the problems from [M1] chapter 9.
- Do the following problems and exercises from [M2] chapter 2 - 2.4, 2.6, 2.13, 2.14, 2.17, 2.18, 2.22, 2.24, 2.31, 2.32, 2.35, 2.38. If you have more time, do the remaining problems and exercises from chapter 2.

References

[M1] Morin, David J. *Problems and Solutions in Introductory Mechanics.*

[M2] Morin, David J. *Introduction to Classical Mechanics: With Problems and Solutions.*