Harvard Physics Circle

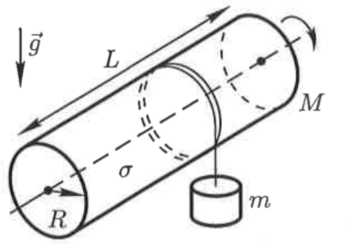
Various Problems II

Sabina Dragoi

§1 Movement in mixed systems

§1.1 Rotation of charged cylinder - Russian Olympiad Final Stage 2008

A long dielectric thin-wall cylinder of radius R and length $L \gg R$ and a mass M carries an electric charge of uniform surface density $\sigma(C/m^2)$. The cylinder can rotate without friction around its axis under a force exerted by a weight of mass m suspended by a light thread wound around the cylinder. Determine the weight acceleration. The magnetic constant μ_0 is known.



§1.2 Water drop - 200 more physics problems, problem 45

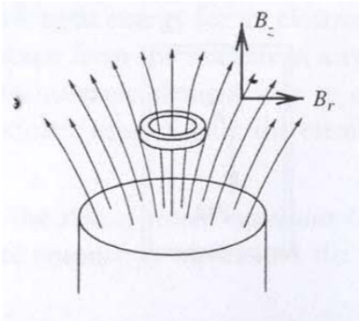
In a very dense fog, there are many tiny water drops that 'float' in the air with negligible speed. If one of the water drops, which is a little larger than the rest, begins to sink, it absorbs those smaller drops that lie in its path (see figure). The ever-growing drop, which can be regarded as spherical, is found to be accelerating uniformly, despite the air drag – proportional to the square of the speed and the cross-sectional area of the drop – acting upon it. What is the maximum possible value for this acceleration?

This is the same problem as the one on Harvard's Physics Department Problems, which you can find at https://www.physics.harvard.edu/files/prob5.pdf and the solution is here https://www.physics.harvard.edu/files/sol5.pdf.

§1.3 Levitating ring - 200 Puzzling Problems, problem 182

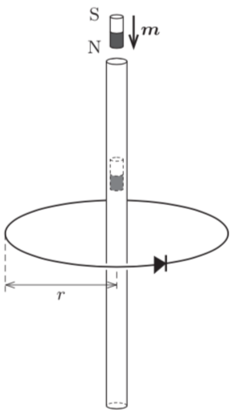
A thin superconducting ring (zero resistance) is held above a vertical, cylindrical magnetic rod, as shown in the figure. The cylindrically symmetrical magnetic field around the ring can be described approximately in terms of the vertical and radial components of the magnetic field vector as $B_z = B_0(1 - \alpha z)$ and $B_r = B_0\beta r$, where B_0, α, β are constants and z and r represent the vertical, and respectively radial coordinates. Initially, the ring has no current flowing through it. When released, it starts moving downwards, while maintaining its symmetry axis vertical. Using the data below, determine how the ring moves subsequently. What current flows through the ring?

Assume the initial coordinates are $z = r = 0$. We also know the following properties about the ring: $m = 50\text{mg}$, $r_0 = 0.5\text{cm}$, $L = 1.3 \times 10^{-8}\text{H}$. The given constants are $B_0 = 0.01\text{T}$, $\alpha = 2m^{-1}$, $\beta = 32m^{-1}$.



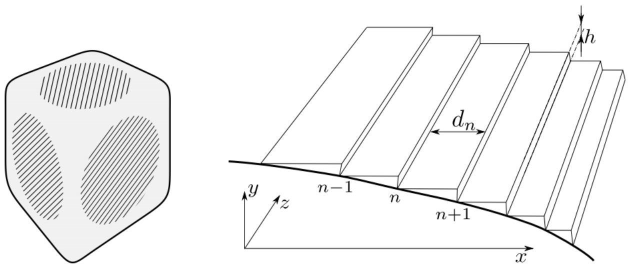
§1.4 Falling magnet through glass tube - 200 more physics problems, problem 180

A closed circular loop of radius r is made from a wire of resistance R and a diode, which can be considered ideal. The loop is held in a horizontal plane, and a long, vertical glass tube passes through its centre (see figure). Find the charge that flows through the diode if a small bar magnet with magnetic moment m falls through the tube.



§1.5 Crystal physics (Staircase) - EuPhO 2018, Problem 3

The equilibrium shape of bodies in zero-gravity is determined by the minimum of their surface energy. Thus, for example, the equilibrium shape of a water droplet turns out to be spherical: the sphere has the smallest surface area among bodies of the same volume. At low temperature, the equilibrium shape of crystals may have flat facets. The parts of the crystal surface that have a small angle ϕ with the facet are in fact staircases of rare steps on this facet. The height of such steps is equal to the period of the crystal lattice h. Equilibrium surface profile $y(x)$ of a certain crystal and the corresponding microscopic staircase are shown schematically in the figure, where n denotes the step number, counting from $x = 0$. The profile shape at x > 0 can be approximated as $y(x) = -(x/\lambda)3/2h$, where $\lambda = 45\mu\text{m}$ and $h = 0.3\text{nm}$.



a) Express the distance d_n between two adjacent steps as a function of n for $n \gg 1$.

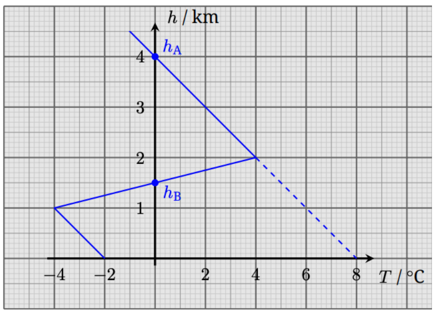
b) The interaction energy E of two steps depends on the distance d between them as

$$E(d) = \mu d \nu$$

,where μ is a constant. Assume that only adjacent steps interact. Find the numerical value for the exponent ν.

§1.6 Ice melting due to friction with air (Ice pellets) - EuPhO 2019, Problem 1

An interesting weather phenomenon can occur when the temperature profile in the atmosphere shows an inversion. The solid blue line in figure 1 shows such a temperature profile. The inversion occurs at heights between 1 km and 2 km. Under these conditions snow falling through the atmosphere (partially) melts in the warmer layer and (partially) freezes again before reaching the ground in the form of "ice pellets". Assume that a small, spherical ice droplet almost completely melts while falling through the atmospheric layer between h_A and h_B where the temperature is above freezing point.



1. Determine the mass fraction of the droplet that freezes before reaching the ground.

2. Find, as precisely as possible, the temperature of the droplet at ground level if there were no inversion and the temperature profile followed the dashed line below a height of 2 km.

§1.7 Error estimation - F=ma 2019B, Problem 25

A student makes an estimate of the acceleration due to gravity, g, by dropping a rock from a known height h and measuring the time, t, it takes to hit the ground. Neglecting air resistance, which one of the following situations will lead to the smallest value of the relative uncertainty, $(\Delta g)/g$, in the estimate?

(A) There is no uncertainty in t, and h has a 10% uncertainty.

(B) There is no uncertainty in h, and t has a 10% uncertainty.

(C) Both t and h have a 5% uncertainty.

(D) (A) and (B) yield the same uncertainty, which is smaller than in (C).

(E) (A), (B), and (C) all yield the same uncertainty.

§2 Practice Problems

1. APhO 2012, Problem 1 - Drag on falling magnet (similar to problem 1.4), can be found here: http://staff.ustc.edu.cn/ bjye/ye/APhO/2012Q1.pdf , and its solution is here: http://staff.ustc.edu.cn/ bjye/ye/APhO/2012S1.pdf

2. IPhO 2012, Problem 2 - Coat Hanger (an oscillation problem), can be found here along with its solution: https://www.ioc.ee/ kalda/ipho/13th_IPhO_1982.pdf

3. APhO 2005, Problem 1A - Spring Cylinder with massive piston (an oscillation problem that also includes thermodynamics), can be found here along with its solution: http://mpec.sc.mahidol.ac.th/apho10/sites/default/files/APhO2005_theo_sol.pdf

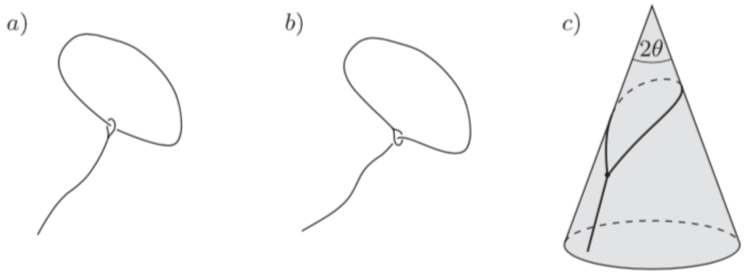
4. EuPhO 2018, Problem 1 - Three balls (unique approach for a purely mechanical system), can be found here: https://www.ioc.ee/ kalda/ipho/2EuPhO/eupho18-theory.pdf, with the solution here http://eupho2018.mipt.ru/pdf/eupho18-th-solution.pdf

5. Harvard Problem of the Week 9 - Fractal Moment of Inertia (nice idea, purely mechanical), can be found here: https://www.physics.harvard.edu/undergrad/problems

§2.1 Lasso over an iceberg - 200 more physics problems, problem 81

This is a purely mechanical problem, but with an interesting, simple, geometric approach. The committee of the Glacier Climbing Club has decided to introduce a new challenge for its members: they have to climb as high as they can on an artificial right-circular cone, the surface of which has been made into a very smooth, slippery 'iceberg', by letting water trickle down it in sub-zero temperatures. A lasso is to be their only piece of climbing equipment!

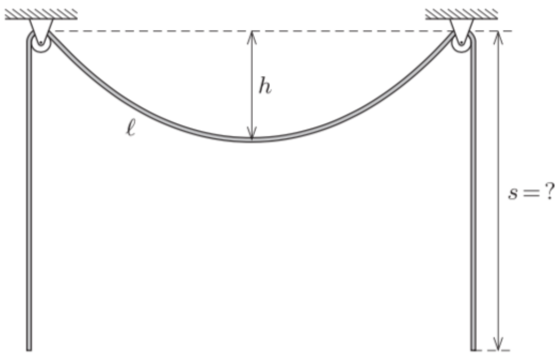
The lasso for novices is as shown in figure a) and consists of a length of rope attached by a small eyelet to a closed loop of fixed length. That for experts is shown in figure b) and is a single rope with, at one end, an eyelet, through which the other end is threaded. All the ropes are light compared to the mass of a climber, and friction between them and the ice, and within the eyelet, is negligible.



For what ranges of cone angle, 2θ , are (i) the novices and (ii) the experts able to climb up the iceberg using their lassos in the way illustrated in figure c)? Assume that the straight segment of a lasso follows one of the cone's lines of steepest descent.

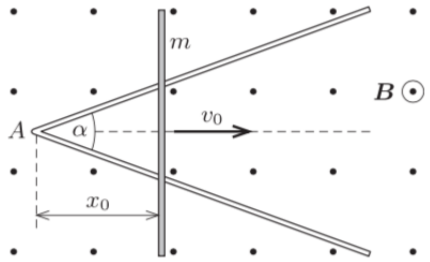
§2.2 Rope over 2 pulleys - 200 more physics problems, problem 78

Again, this is a purely mechanical problem, but it has a very elegant and worth noting solution. A uniform flexible rope passes over two small frictionless pulleys mounted at the same height (see figure). The length of rope between the pulleys is l, and its 'sag' is h. In equilibrium, what is the length s of the rope segments that hang down on either side?



§2.3 Rod in magnetic field - 200 more physics problems, problem 177

A long, straight wire of negligible resistance is bent into a V shape, its two arms making an angle α with each other, and placed horizontally in a vertical, homogeneous magnetic field of strength B. A rod of total mass m, and resistance r per unit length, is placed on the V-shaped conductor, at a distance x_0 from its vertex A, and perpendicular to the bisector of the angle α. The rod is started off with an initial velocity v_0 in the direction of the bisector, and away from A. The rod is long enough not to fall off the wire during the subsequent motion, and the electrical contact between the two is good – although the friction between them is negligible. Where does the rod ultimately stop?


m
B
A
α
v_0
x_0